As an illustration of the results of the calculations, assume η lies between 0.6 and 0.8 and $h_c'/h_g = 1.0$. With these data and referring to Fig. 1, it can be seen that the reduction in the film-coolant flowrate, as a result of the combined use of regenerative and gaseous film cooling, will be between 30 and 55%. These results are based upon $\phi = 0.1$, which experiments have indicated to be a typical value. Because a completely film-cooled chamber is of relatively simple construction and consequently is potentially capable of high reliability, the previously mentioned reduction of film-coolant flow must be weighted against the increase in complexity if the regenerative cooling system is added.

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Gaseous Film Cooling with Multiple Injection Stations

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Nomenclature

A_s = cooled surface area
c_{pc} = specific heat of coolant
d = wall thickness
h_c = coolant-side heat transfer coefficient
h_c' = $1/(1/h_c + d/k)$
h_g = gas-side heat transfer coefficient without film cooling
k = thermal conductivity of the wall
n = number of slots
s = distance downstream of last slot
T_{ad} = adiabatic wall temperature without film cooling
T_{ad}' = adiabatic wall temperature with film cooling
T_c = temperature of coolant at exit of film-coolant injector
T_{wg} = gas-side wall temperature
V_c = velocity of coolant at exit of film-coolant injector
V_g = velocity of gas at the point where the film coolant is introduced
$\dot{w}_c$ = coolant flow rate
ϵ = dimensionless measure of distance from film coolant injector where wall temperature equals T_c
η = defined by Eq. (2a)
η^* = defined by Eq. (1a)
ϕ = efficiency factor
ψ = efficiency factor

When wall temperatures are lowered by using gaseous film cooling, it is usually necessary or wise to inject the coolant along the wall through a series of openings in the wall.

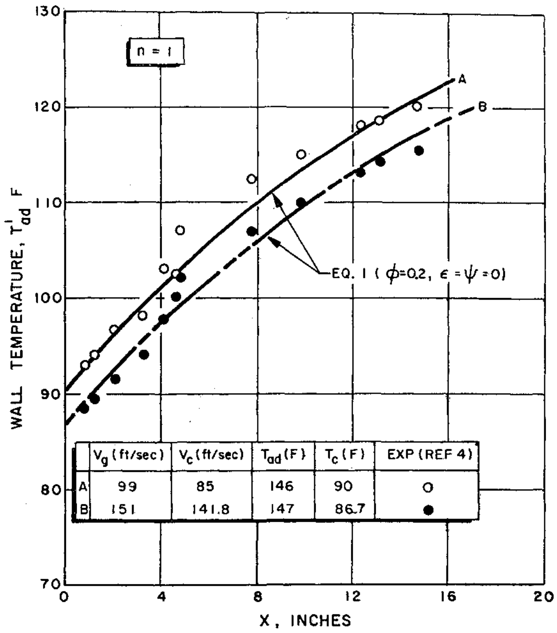

Fig. 1 Wall temperature vs distance downstream of slot, $n = 1$.

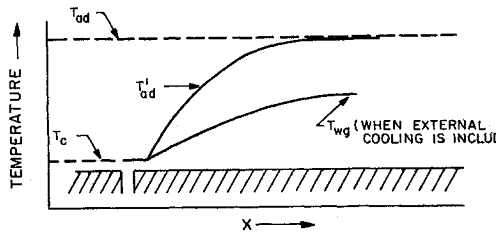

Fig. 2 Adiabatic wall temperature, $n = 1$.

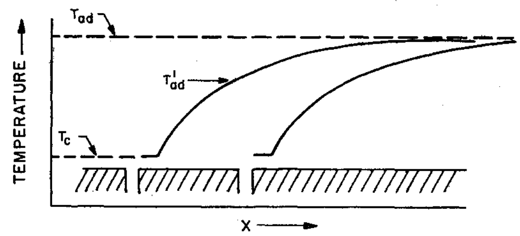

Fig. 3 Adiabatic wall temperatures, $n = 2$.

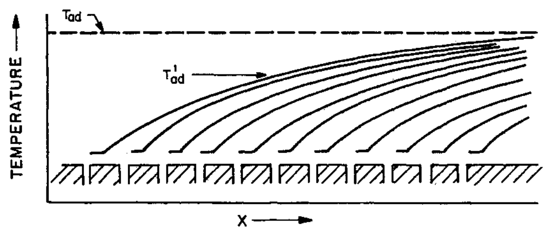

Fig. 4 Adiabatic wall temperature, $n = 11$.

In the establishment of an analytical model for the multiple injection case, one would expect that it is somewhat more complicated than when the coolant is injected at only one station. The coolant introduced at a particular station will not only affect the temperature of the wall immediately downstream of its injection point, but its influence will extend by a progressively lessening degree beyond successive film-coolant injection stations.

The thermal protection achieved downstream from coolant injected at a single injection station has been determined experimentally for a wide range of coolant-gas conditions, but

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there is a scarcity of data for injection with more than one station under similar conditions.

The purpose of this note is to point out with the help of previously published data that, if one uses the concept of an adiabatic wall temperature with film cooling as originally proposed by Eckert,[1] the multiple injection case does not present any special difficulty, and data obtained from a single injection station are applicable. In view of the previously mentioned lack of experimental data, this last feature can be of considerable practical importance.

The following equation for the adiabatic wall film-cooling effectiveness was derived in Ref. 2:

$$\eta^* = \exp\left(\frac{-A_s h_g}{\phi c_{pc} \dot{w}_c} + \frac{\epsilon}{\phi} - \psi\right) \tag{1}$$

where

$$\eta^* \equiv \frac{T_{ad} - T_{ad}'}{T_{ad} - T_c} \tag{1a}$$

This equation was extended to include heat transfer (e.g., external regenerative cooling) in Ref. 3. Thus,

$$\eta = \left[\frac{1}{1 + (h_c'/h_g)}\right]\left[\exp\left(\frac{-A_s h_g}{\phi c_{pc} \dot{w}_c} + \frac{\epsilon}{\phi} - \psi\right) + \frac{h_c'}{h_g}\right] \tag{2}$$

where

$$\eta \equiv \frac{T_{ad} - T_{wg}}{T_{ad} - T_c} \tag{2a}$$

Single-Slot Correlation

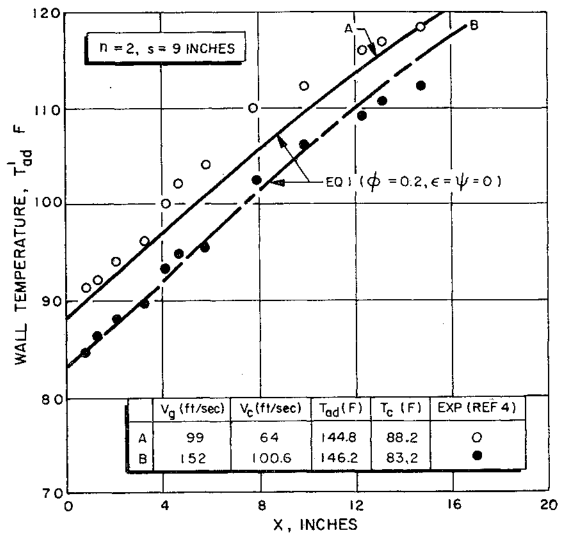
With apparatus featuring a nearly tangential continuous slot for injecting a gaseous coolant (air) between air that was heated and a wall, the authors of Ref. 4 obtained single-slot (and multiple-slot) adiabatic wall temperature data. It should be pointed out, however, that these experiments were conducted at low gas velocities and temperatures. A small portion of their single-slot data is presented in Fig. 1, along with a curve of the NASA correlation [Eq. (1)] that correlated the data quite well when $\phi = 0.2$ and $\epsilon = \psi = 0$. The data chosen corresponded to runs in which $h_g \simeq$ const and $V_c/V_g \simeq 1.0$.

Multiple-Slot Correlation

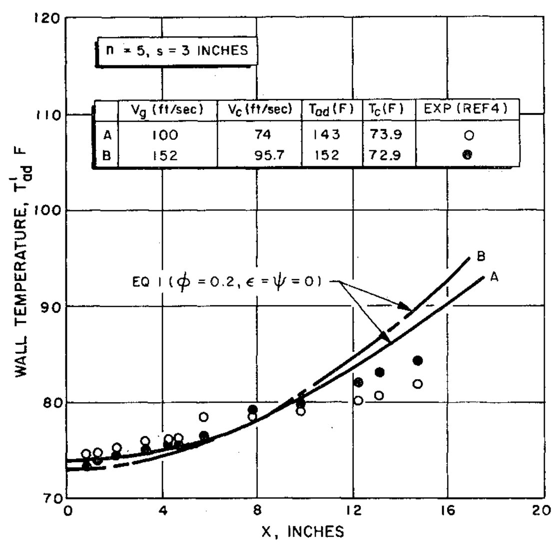
The multiple-slot correlation is perhaps best explained by first referring again to a single slot (Fig. 2). The adiabatic wall temperature with film cooling, T_{ad}', can be determined as discussed previously from Eq. (1) and is shown in Fig. 3. Now, assume that a new slot is introduced downstream. Again, the adiabatic wall temperature can be determined from Eq. (1), but note that the T_{ad} is a function of x and is indeed the T_{ad}' curve from the original slot. The procedure is repeated if additional slots are added (Fig. 4), keeping in mind that T_{ad} is different for each slot and each time is equal to the T_{ad}' results from the preceding slot.

Using the foregoing concept of a progressively decreasing adiabatic wall temperature, Eq. (1) was compared with the multiple-slot experimental data of Ref. 4 in Fig. 5. Unfortunately, wall temperatures were taken only downstream of the last slot in each case. The calculated adiabatic wall temperature curves for the $n = 5$, $s = 3$-in. run (Fig. 5b) are presented in Fig. 6 and demonstrate the overlapping thermal effect of multiple slots. The rather good agreement between the experimental and calculated wall temperatures for $n = 2$ and 5 in Fig. 5† is considered to be supporting evidence that the multiple-slot case can be analyzed by using the concept of an adiabatic wall temperature as described in the foregoing. The procedure has been used with some

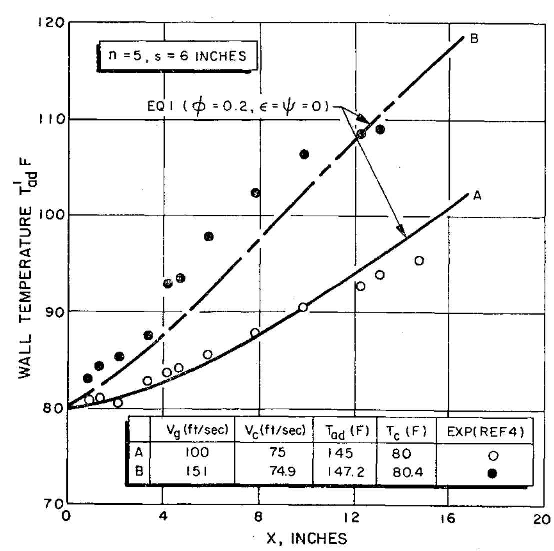
† Although they were not included in Fig. 5 to conserve space, favorable comparisons with the experimental data were found for $n = 2, s = 3$ and 6 in.; $n = 3, s = 6$ in.; and $n = 4, s = 9$ in.



a) $n = 2$, $s = 9$ in.



b) $n = 5$, $s = 3$ in.



c) $n = 5$, $s = 6$ in.

Fig. 5 Wall temperature vs distance downstream of last slot.

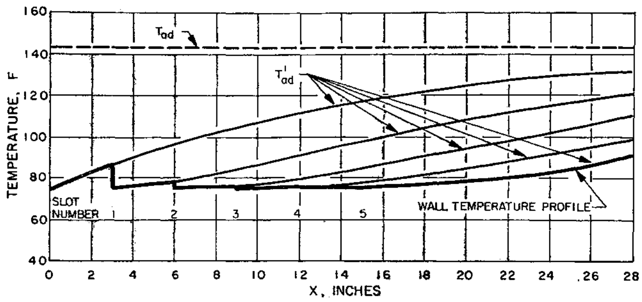


Fig. 6 Calculated temperature profiles for run where n = 5 and s = 3 in.

success in correlating gaseous film-cooling data obtained under full-scale rocket engine conditions.

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Similar and Nonsimilar Solutions of the Nonequilibrium Laminar Boundary Layer

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Nomenclature

c_i = mass fraction of species i
c_{pi} = specific heat of species i, ft²/(sec²-°R)
$\bar{c}_p$ = specific heat of the mixture, ft²/(sec²-°R)
f = stream function variable defining velocity, $f = \int (u/u_e)d\eta$
h_i = static enthalpy of species i, (ft/sec)²
Le = Lewis-Semenov number
p = static pressure, psf
Pr = Prandtl number
R = universal gas constant, psf/(lb-mole-sec²-°R)
T = absolute temperature, °R
u = component of velocity parallel to surface, fps
$\dot{w}_A$ = net rate of production of atoms, slug/ft³/sec
x = distance along body surface, ft
β = pressure gradient parameter, $(2x/u_e)(du_e/dx)$
θ = temperature ratio, T/T_e
ρ = gas density, slug/ft³
η = nondimensional coordinate normal to body surface, $\eta = [u_e/2x(\rho\mu)_r]^{1/2}\int \rho dy$

Subscripts

A = atoms
e = at the edge of the boundary layer
eq = at local equilibrium
M = molecules
w = at the wall
A prime denotes differentiation with respect to η.

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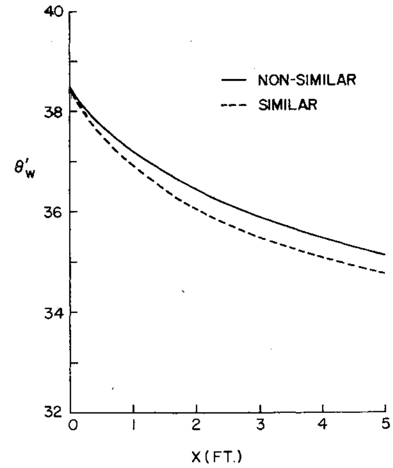

Fig. 1 Temperature gradient at the wall.

The results of nonsimilar laminar boundary-layer flow with finite chemical reactions along a wall with zero pressure gradient are reported. In addition, the validity of employing the concept of local similarity for chemical nonequilibrium flow is examined by a direct comparison of the nonsimilar solution with a locally similar one.

The results are for a binary mixture (oxygen is this note) with concentration diffusion, a constant Lewis-Semenov number of 1.4, and a constant Prandtl number of 0.7. In the numerical results the specific heat of the mixture, $\bar{c}_p = c_A(c_{p_A} - c_{p_M}) + c_{p_M}$, is taken as a constant with the same value as the specific heat of the atoms, c_{p_A}, and the molecules, c_{p_M}. With the nondimensional density-viscosity product equal to one, the momentum, energy, and conservation of atoms equations for local similarity become

$$f''' + ff'' = 0$$

$$\frac{\theta''}{Pr} + f\theta' + \frac{u_e^2}{\bar{c}_p T_e}(f'')^2 - \frac{2x}{u_e}\left(\frac{\dot{w}_A}{\rho}\right)\frac{(h_A - h_M)}{\bar{c}_p T_e} = 0$$

$$\frac{Le}{Pr}C_A'' + fC_A' + \frac{2x}{u_e}\left(\frac{\dot{w}_A}{\rho}\right) = 0$$

where

$$\frac{\dot{w}_A}{\rho} = -\frac{3.39 \times 10^{22}}{(T_e\theta)^4}\left(\frac{p_e}{R}\right)^2 \times \left[\frac{C_A^2}{1 + C_A} - 2116.216\,(1 - C_A)\exp\left(15.8 - \frac{108{,}000}{T}\right)\right]$$

The boundary conditions at a catalytic wall with specified temperature are $f(0) = 0$, $f'(0) = 0$, $\theta(0) = \theta_w$, $C_A(0) = C_{A_{eq}}(\theta_w, p_e)$ and at the outer edge are $f'(\eta_e) = 1$, $\theta(\eta_e) = 1$, and $C_A(\eta_e) = C_{A_e}$.

Locally similar solutions are obtained by solving the forementioned set of ordinary differential equations with two-

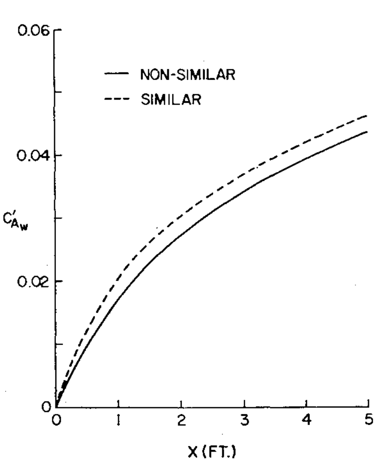

Fig. 2 Atom mass fraction gradient at the wall.